

## Unit 5 – Random Variables

### Guided Notes

#### B. Binomial Distributions

A binomial setting arises when we perform  $n$  independent trials of the same chance process and count the number of times that a particular outcome (called a "success") occurs.

The four conditions for a binomial setting are

- Binary? The possible outcomes of each trial can be classified as "success" or "failure."
- Independent? Trials must be independent. That is, knowing the result of one trial must not tell us anything about the result of any other trial.
- Number? The number of trials  $n$  of the chance process must be fixed in advance.
- Same probability? There is the same probability of success  $p$  on each trial.

**6. Football?** Professional football teams in the NFL must play 16 regular season games. Each game begins with a coin toss to determine who gains possession of the ball first. Suppose one team is chosen and you record the number of times they win the coin toss in one season. Is this a binomial setting?

- **Binary?** "Success" = wins the coin toss. "Failure" = doesn't win the coin toss.
- **Independent?** Knowing the outcome of one coin toss tells you nothing about the outcome of another coin toss.
- **Number?**  $n = 16$
- **Same probability?**  $p = 0.50$

**This is a binomial setting.**

The count of successes  $X$  in a binomial setting is a binomial random variable. The possible values of  $X$  are  $0, 1, 2, \dots, n$ .

#### Binomial Probability Formula

Suppose that  $X$  is a binomial random variable with  $n$  trials and probability  $p$  of success on each trial. The probability of getting exactly  $k$  successes in  $n$  trials ( $k = 0, 1, 2, \dots, n$ ) is

$$P(X = k) = {}_n C_k \cdot p^k (1 - p)^{n-k} \quad \text{Where} \quad {}_n C_k = \frac{n!}{k!(n-k)!}$$

The binomial probability formula looks complicated, but it is fairly straightforward if you know what each part of the formula represents:

$$P(X = k) = (\# \text{ of ways to get } k \text{ successes in } n \text{ trials}) \cdot (\text{success probability})^k (\text{failure probability})^{n-k}$$

**7. Who won the toss?** Professional football teams in the NFL must play 16 regular season games. Each game begins with a coin toss to determine who gains possession of the ball first. What is the probability that one team wins the coin toss in exactly 13 of their 16 games?

Let  $X$  = the number of coin tosses won in a season.  $X$  is a binomial random variable with  $n = 16$

$$P(X = 13) = {}_{16}C_{13} \cdot (0.5)^{13} \cdot (1 - 0.5)^{16-13}$$

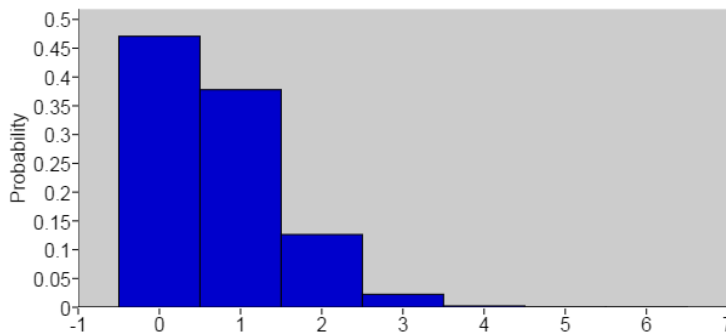
and  $p = 0.50$ .

$$= 560 \cdot (0.5)^{13} \cdot (0.5)^3$$

$$= 0.0085$$

The probability distribution of a binomial random variable is a binomial distribution. Any binomial distribution is completely specified by two numbers: the number of trials  $n$  of the chance process and the probability  $p$  of success on any trial.

**8. What degrees do Americans hold?** According to recent government statistics, 11.8% of all Americans aged 25 or over hold a Master's degree, Professional degree, or Doctoral degree. Randomly select 6 Americans aged 25 or over and let  $X$  = the number (out of these 6) that hold such a degree. Make a histogram of the probability distribution of  $X$ . Describe its shape.



S - The graph is skewed right with a single peak at  $X = 0$ . (Notice that  $X = 4$ ,  $X = 5$ , and  $X = 6$  have such low probabilities that their bars cannot be seen on this scale.)

O - none

C - about 1

S - the number of degrees held by the adult out of 6 ranges from 0 to 6

### Calculating the Mean of a Binomial Random Variable

If a count  $X$  of successes has a binomial distribution with number of trials  $n$  and probability of success  $p$ , the mean of  $X$  is  $\mu_x = np$

### Calculating the Standard Deviation of a Binomial Random Variable

If a count  $X$  of successes has a binomial distribution with number of trials  $n$  and probability of success  $p$ , the standard deviation of  $X$  is  $\sigma_x = \sqrt{np(1-p)}$

**9. Do you have loops, whorls, or arches?** The fingerprint of every finger can be classified into one of three basic patterns: a loop, a whorl, or an arch. Forensic experts have determined that about 65% of all fingers have loops, 30% have whorls, and 5% have arches. Suppose that these percentages are exactly correct and patterns are independent from one finger to the next. Select a person at random and classify each of their fingerprints on both hands. Let  $X$  = the number of fingerprints with a loop.

(a) Calculate and interpret the mean of  $X$ .

$\mu_x = 10(0.65) = 6.5$  . **If many people were selected, we'd expect 6.5 fingerprints with a loop, on average, on an individual's two hands.**

(b) Calculate and interpret the standard deviation of  $X$ .

$\sigma_x = \sqrt{10(0.65)(0.35)} = 1.508$  **If many people were selected, the number of fingerprints with a loop on an individual's two hands would typically vary from the mean of 6.5 fingerprints by about 1.508 fingerprints.**

**10. Feeling loop-y?** The fingerprint of every finger can be classified into one of three basic patterns: a loop, a whorl, or an arch. Forensic experts have determined that about 65% of all fingers have loops, 30% have whorls, and 5% have arches. Suppose that these percentages are exactly correct and patterns are independent from one finger to the next. Select a person at random and classify each of their fingerprints on both hands. Let  $X$  = the number of fingerprints with a loop. The probability distribution of  $X$  is shown below, with probabilities rounded to the nearest thousandth.

Number of loops	0	1	2	3	4	5	6	7	8	9	10
Probability	0.000	0.001	0.004	0.021	0.069	0.154	0.238	0.252	0.176	0.072	0.013

Victor has 3 loop patterns in his fingerprints and wonders what the probability is to have more than this. Find the probability that a randomly chosen person has 3 or fewer loop patterns.

$$\begin{aligned}
 P(X \leq 3) &= P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) \\
 &= 0.000 + 0.001 + 0.004 + 0.021 \\
 &= 0.026
 \end{aligned}$$

### Cumulative Binomial Probabilities with Technology

The TI-83/84 command  $\text{binomcdf}(n,p,k)$  computes  $P(X \leq k)$  in a setting with  $n$  trials and success probability  $p$  on each trial.

- Press 2nd VARS(DISTR) and choose B. binomcdf(.
- In the dialog box, enter these values: trials:10, p:0.65, x value:3, choose Paste, and then press ENTER.